# QCD vacuum-polarization corrections

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# Modified approach to hadronic corrections

**modified method handles problematic external scales: $m_\mu$, $Q$**

**broad application of method builds confidence in calculations**

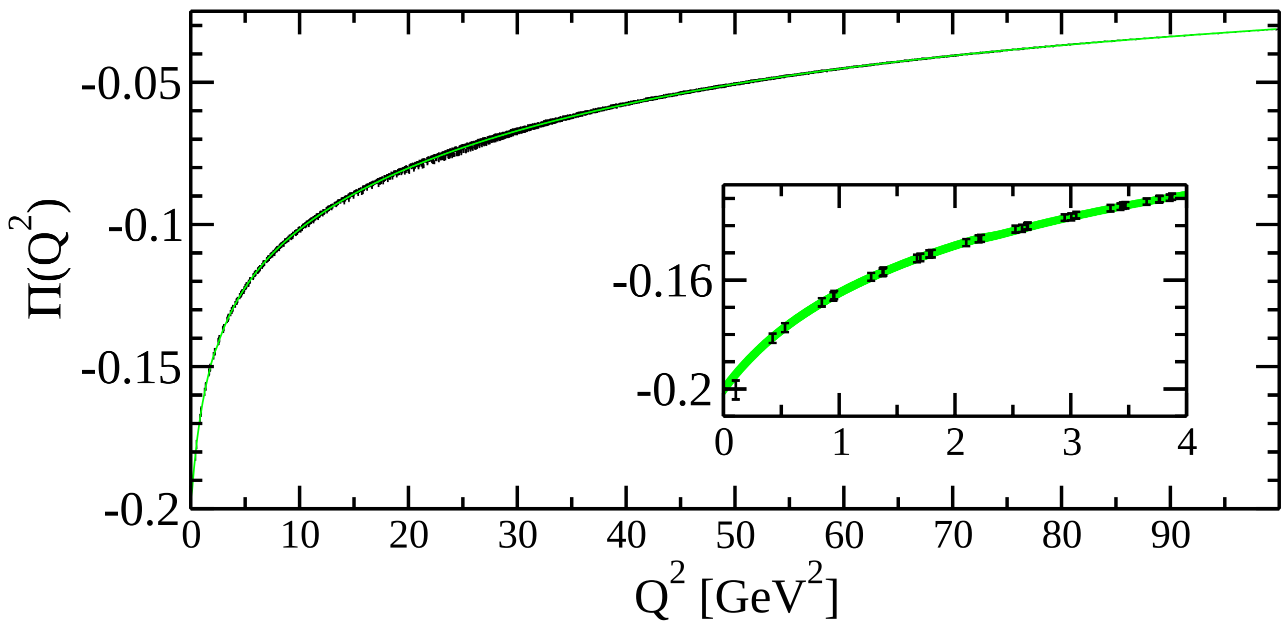
**four-flavor calculations are necessary for high-precision results**

# QCD vacuum-polarization

**focus on corrections due to the QCD vacuum-polarization $\Pi(Q^2)$**


$$= (Q_\mu Q_\nu - Q^2 \delta_{\mu\nu})\Pi(Q^2)$$

**an example of $\Pi(Q^2)$ calculated at a single fixed $a$, $L$ and $m_{PS}$**




**calculation of bare $\Pi(Q^2)$ is relatively simple with lattice QCD**

# A new approach for external scales

**$Q$, an external scale unrelated to QCD, spoils dimensional analysis**

$$\mathit{\Pi}(Q^2) = \mathit{\Pi}_{\mathsf{lat}}\left(a^2 Q^2\right)$$

**appearance of $a$ in a dimensionless observable causes difficulties**

$$\frac{\partial}{\partial a}\left(\mathit{\Pi}(Q^2)\right) \neq 0 \qquad\qquad \mathit{\Pi}(Q^2) \propto g_V^2 \frac{Q^2}{m_V^2}$$

**modified approach eliminates this unwanted scale-dependence**

$$\overline{\mathit{\Pi}}(Q^2) \equiv \mathit{\Pi}\left(\frac{Q^2}{H^2_{\mathsf{phys}}} \cdot H^2\right) \qquad\qquad \lim_{m_{PS}\to m_\pi} \overline{\mathit{\Pi}}(Q^2) = \mathit{\Pi}(Q^2)$$

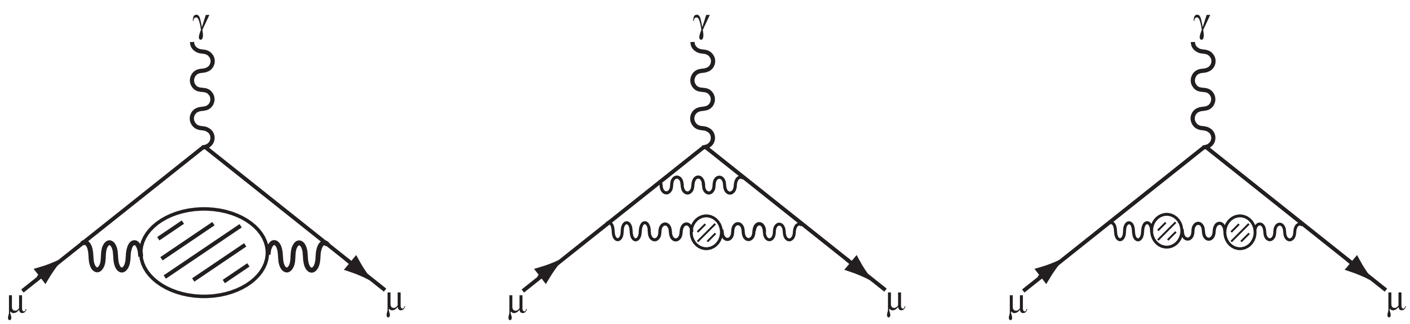
**the choice $H = m_V$ absorbs much of the strong $m_{PS}$ dependence**

# QCD corrections to muon $g-2$

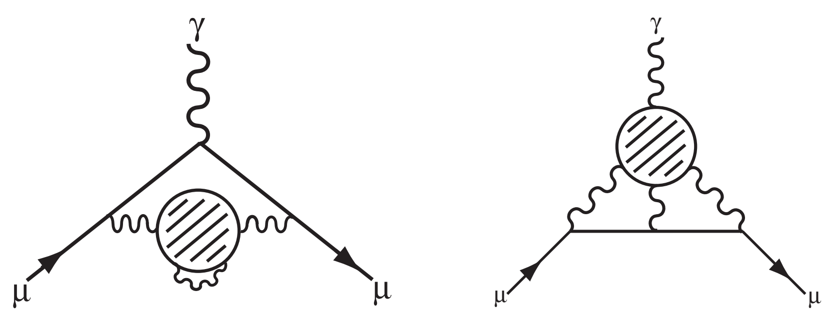
**the QCD piece is a series in $\alpha$ with nonperturbative coefficients**

$$a_\mu^{\text{qcd}} = \sum_{n=2} \alpha^n A_{\mu,\text{qcd}}^{(n)} = a_\mu^{(2)} + a_\mu^{(3,\text{vp})} + a_\mu^{(3,\text{lbl})} + \mathcal{O}(\alpha^4)$$

**we calculated the LO and NLO vacuum-polarization corrections**

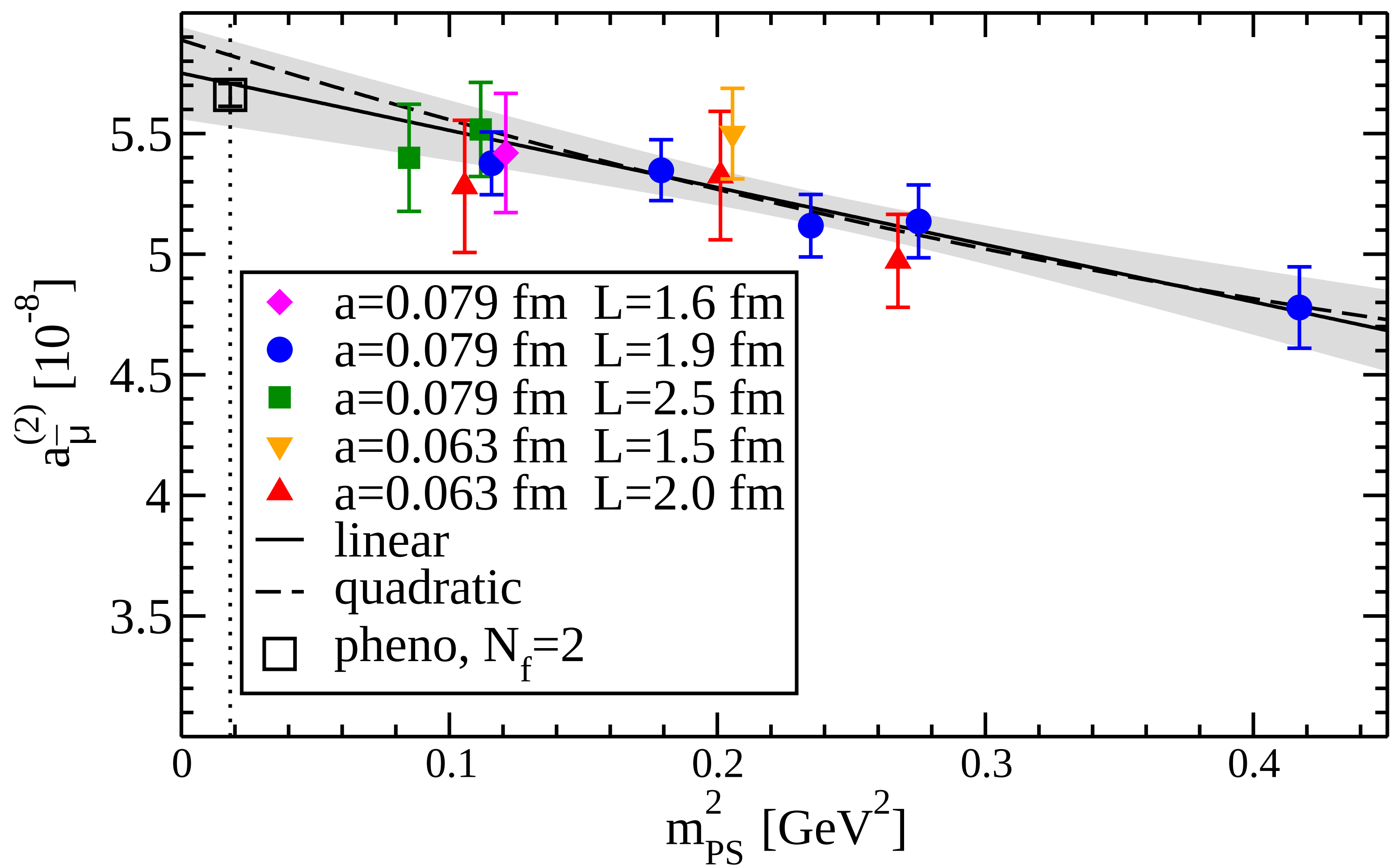



**the NLO light-by-light corrections will not be discussed by me**



# Leading-order correction to $a_\mu$

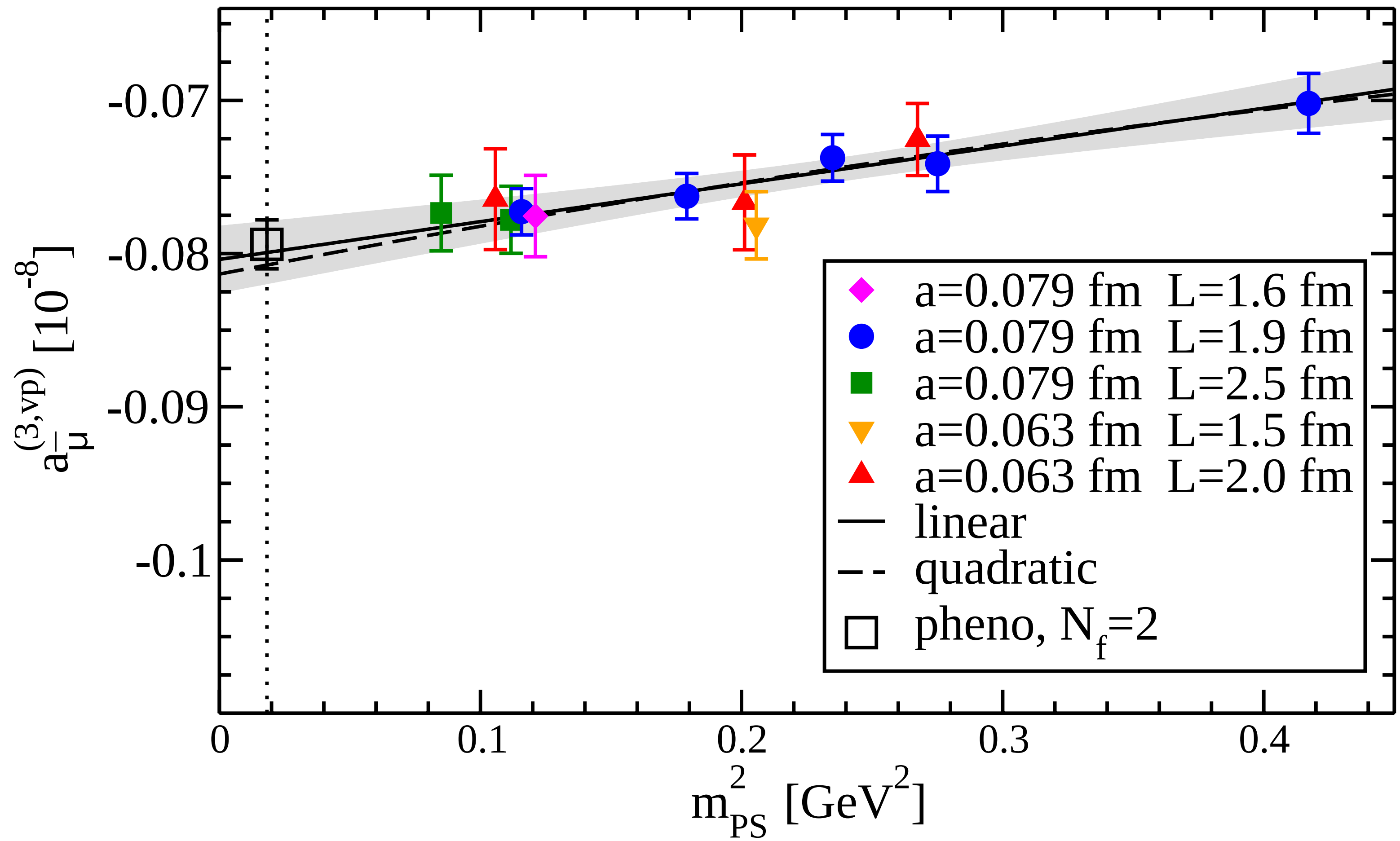
**modified method lead to reliable well-controlled calculation of $a_\mu^{(2)}$**




**use of $N_f = 2$ was the only substantially weak part of calculation**

# Partial higher-order correction to $a_\mu$

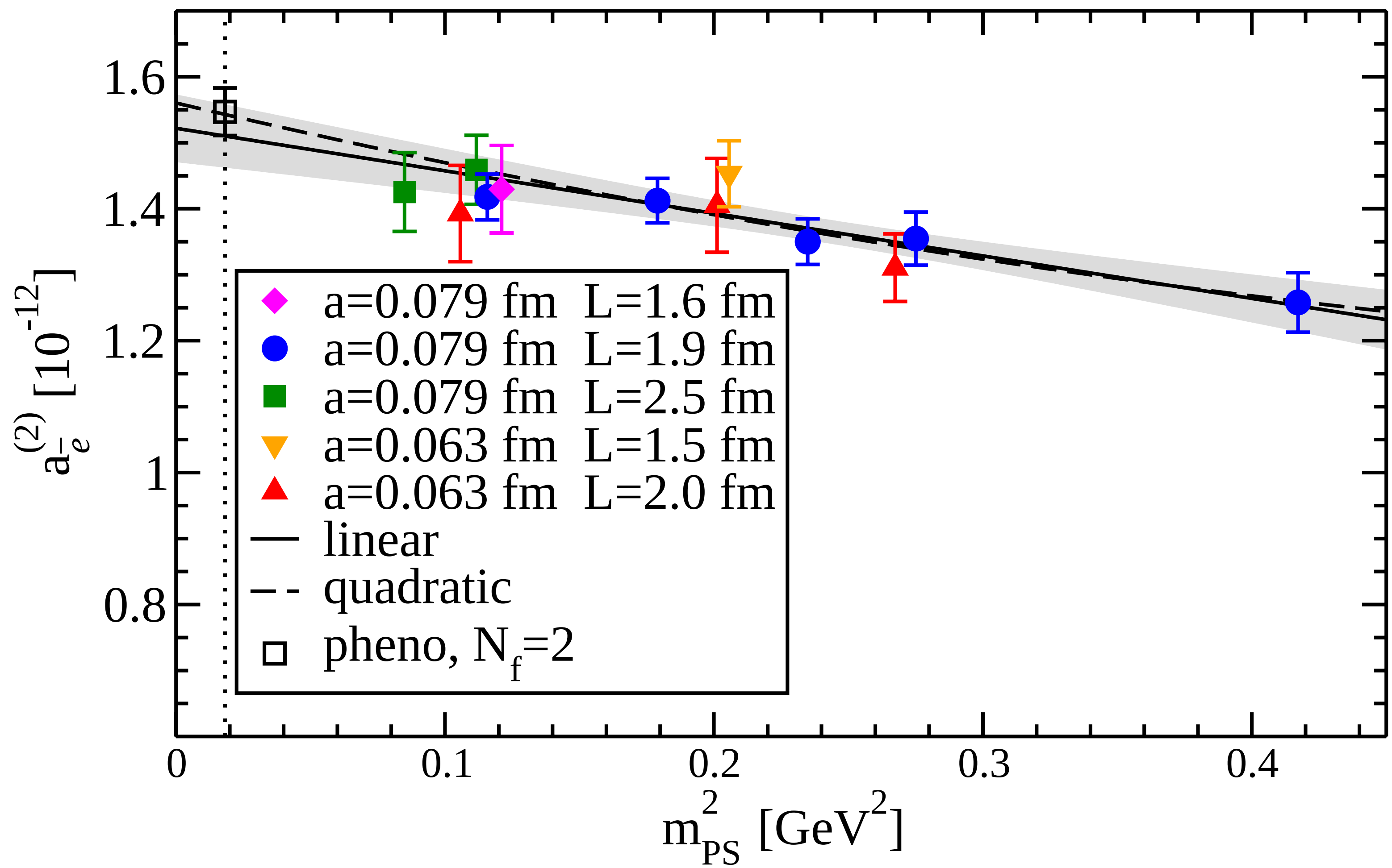
$a_\mu^{(3,\mathrm{vp})}$ is the non-light-by-light portion of the NLO correction




precision comparable to dispersive analysis, sufficient for new exp.

# Leading-order correction to $a_e$

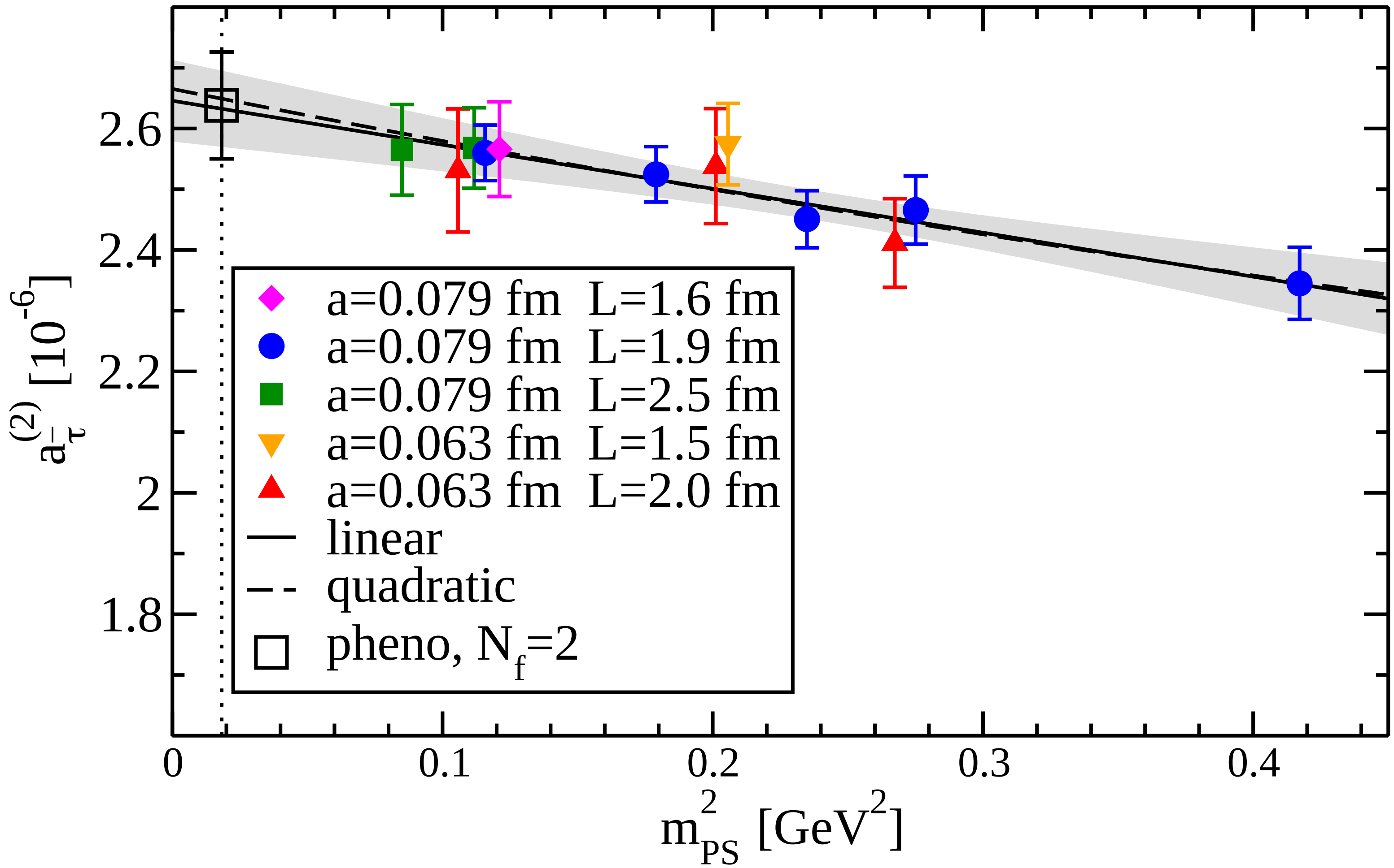
**the measurement of $a_e$ is used to determine the value of $\alpha$**




**measurement of $a_e$ so precise that $\alpha$ is now sensitive to $a_e^{(2)}$**

# Leading-order correction to $a_\tau$

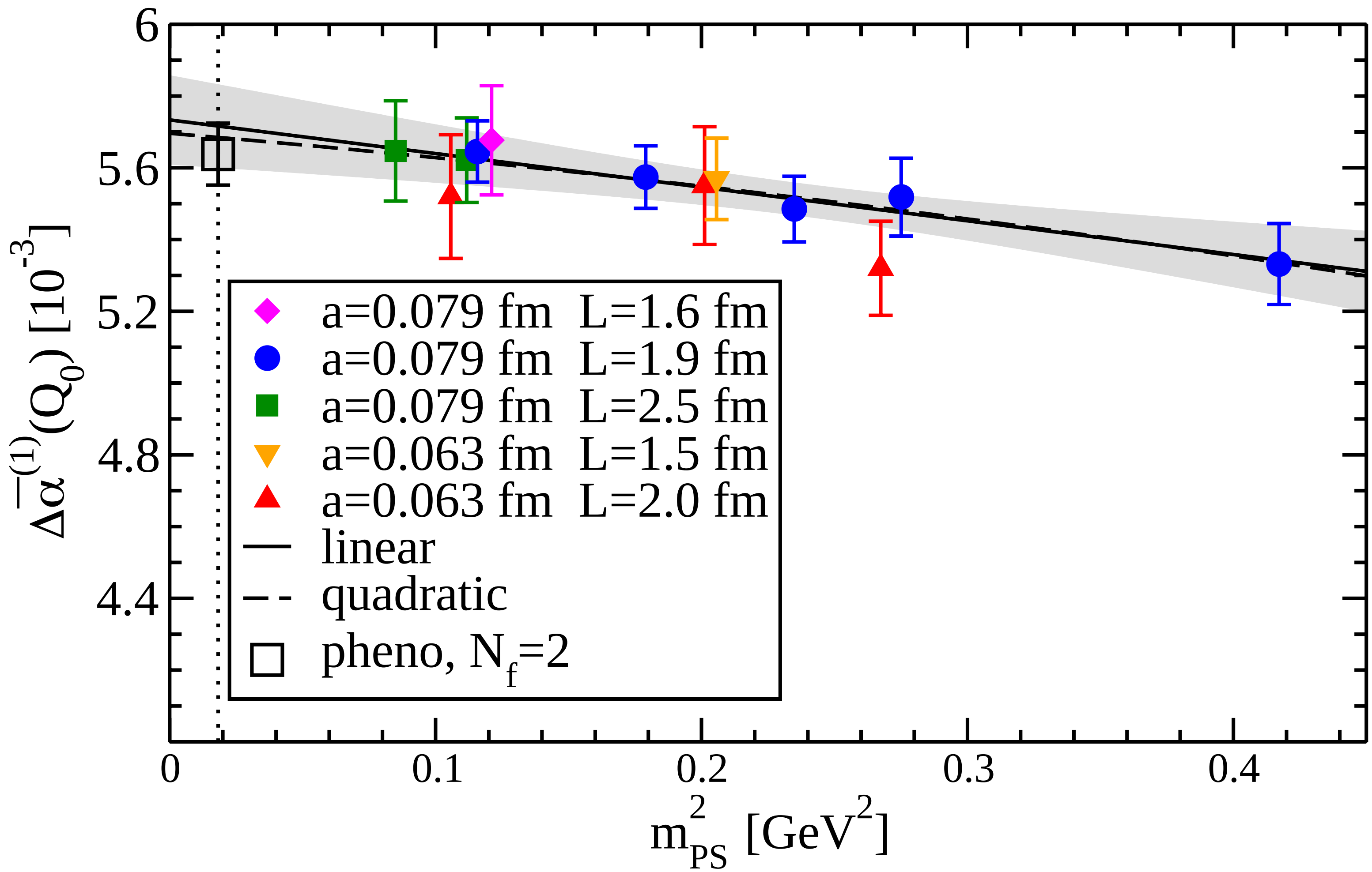
**currently, only bounds on $a_\tau$ are available from experiment**




**however, $a_\tau$ should be much more sensitive to new physics**

# Leading-order correction to $\alpha(Q^2)$

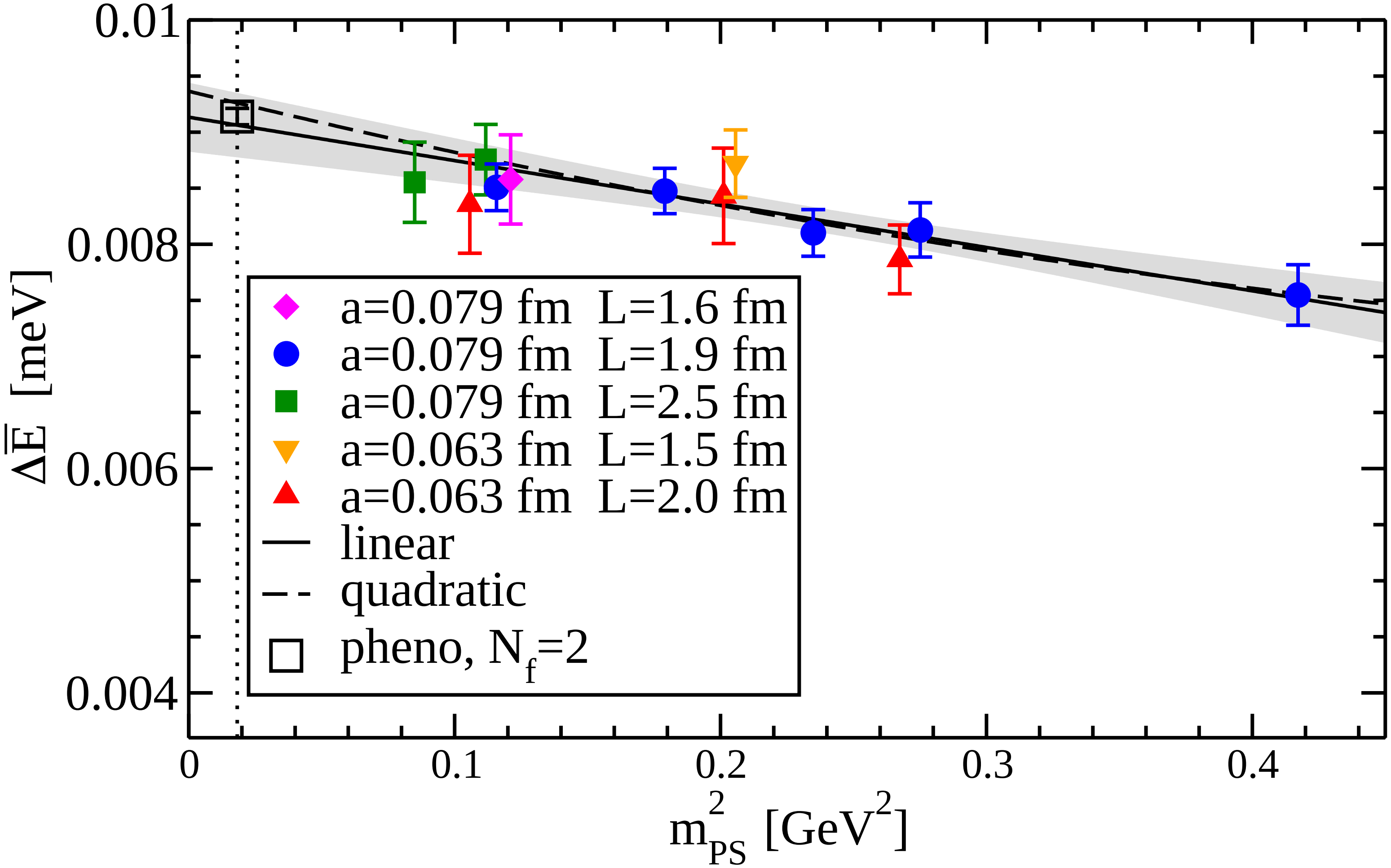
**QCD corrections to running $\alpha(Q^2)$ impact high-energy predictions**




**combining LQCD and PQCD gives $\alpha(M_Z^2)$ completely from theory**

## Leading-order correction to muonic-hydrogen

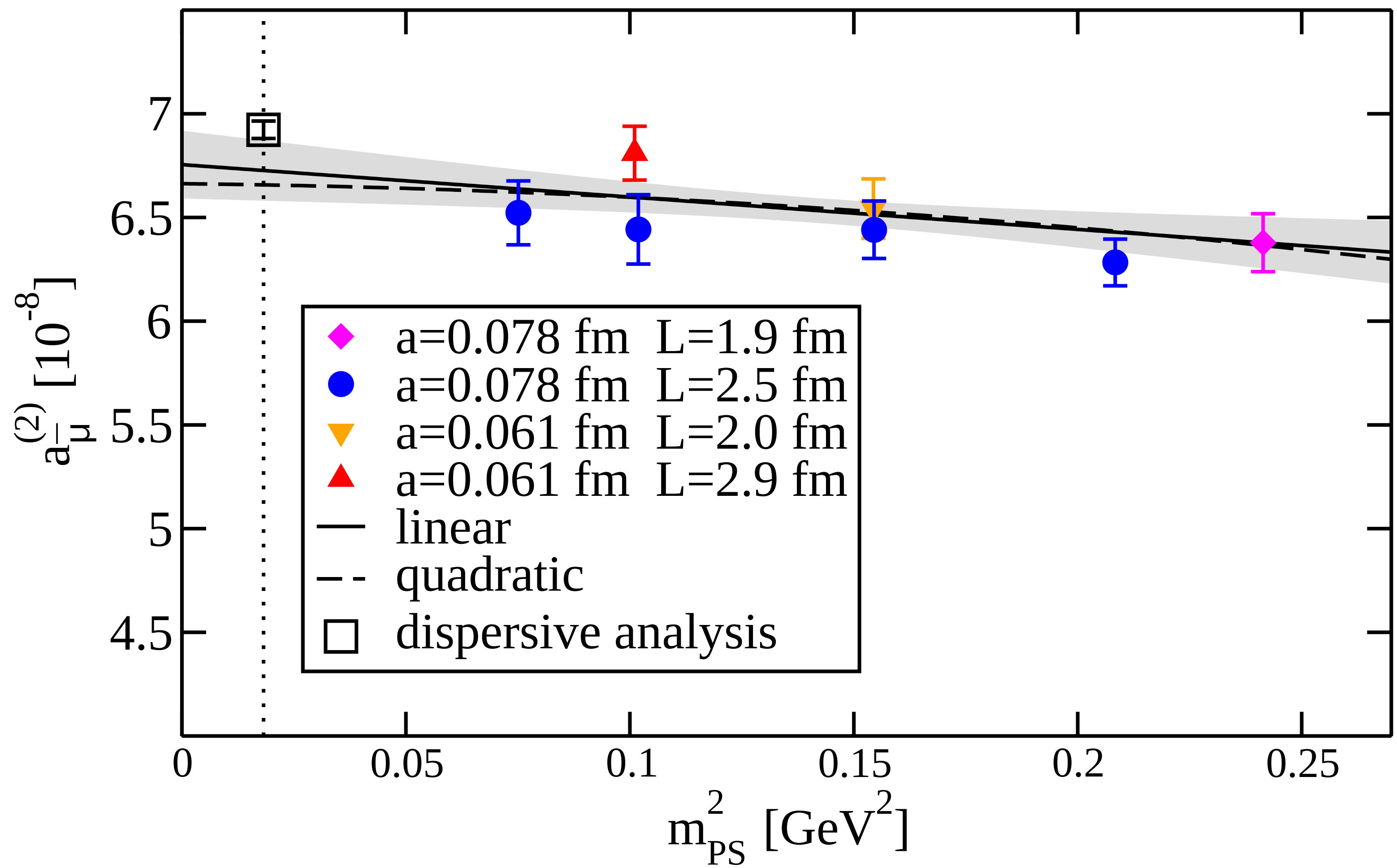
**direct lattice calculation of charge radius is not yet feasible**




**but QCD corrections can be calculated with modified approach**

# Four-flavor calculations for $a_\mu^{(2)}$

**charm contribution is comparable to LBL and EW corrections**




**use of $N_f = 4$ now allows for direct quantitative comparisons**

# Summary

- QCD corrections to EW observables involve external scales
- modified approach handles external scales differently
- new method successfully applied to many quantities in $N_f = 2$
- preliminary $N_f = 4$ result for LO correction $a_\mu^{(2)}$ accurate to $2\%$
- focusing on controlling all uncertainties for $N_f = 4$ calculation
- $N_f = 4$ result for NLO correction $a_\mu^{(3,\mathrm{vp})}$ should be sufficient